

Metric locating domination

versus

Binary locating domination¹

Ignacio M. Pelayo²

[BWIC 2011](#)³

¹J. Cáceres, C. Hernando, M. Mora, L. Moreira, M. L. Puertas.

²UPC, Catalunya, Spain, Europe, The Earth, Solar System, Milky Way.

³[LaBRI , Aquitaine, France, Europe, The Earth, Solar System, Milky Way](#) 

$G = (V, E)$ is a simple finite connected graph.
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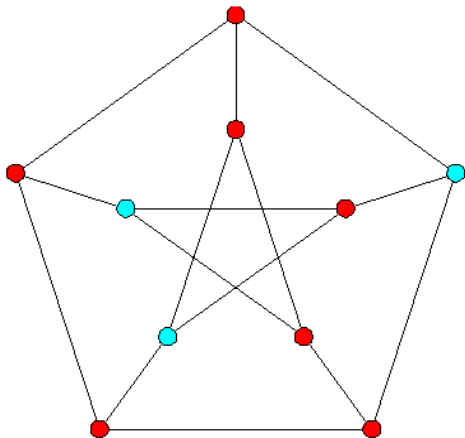
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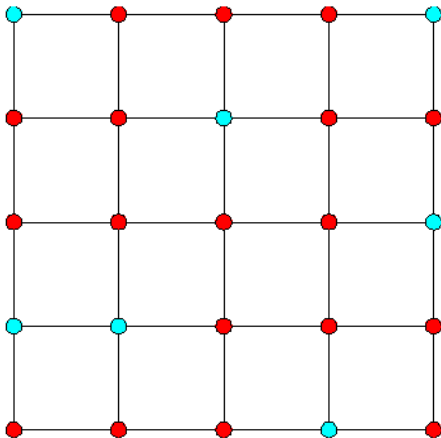
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$\gamma(P) = 3$, since blue vertices form a γ -code.

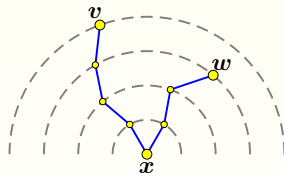


$\gamma(P_5 \square P_5) = 7$, as blue vertices form a γ -code.

Let $G = (V, E)$ be a connected graph and $v, w \in V$.

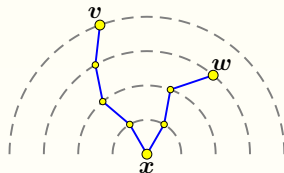
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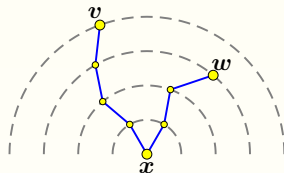
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- $S \subseteq V$ is a *locating set* (also called a *resolving set*) of G if every pair $v, w \in V$ are resolved by some vertex $x \in S$. [SI76, HaMe76]

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- Let $S = \{u_1, \dots, u_k\}$ be a locating set. The ordered set:

$$[d(x, u_1), \dots, d(x, u_k)]$$

is the vector of *metric coordinates* of $x \in V$ w.r.t. S .

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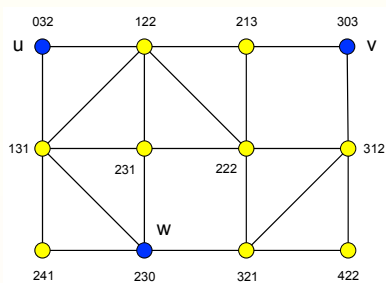
MLD=BLD

TREES

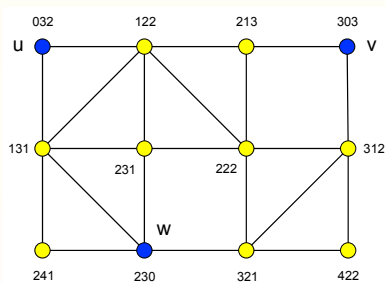
SOME
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- ▷ $\beta(G) = 3$, since $S = \{u, v, w\}$ is a metric basis.
- ▷ Note that vertices **222** and **422** are not dominated by S .

⁴that is, a metric-locating-dominating set.

⁵MLD number for short.

⁶also denoted $\gamma_M(G)$.

- ▷ A set D of vertices in a graph G is an **MLD-set**⁴ if it is both locating and dominating. [HeOe04]

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⇒ Let $S_1, S_2 \subseteq V(G)$. If S_1 is dominating and S_2 is locating, then $S_1 \cup S_2$ is an MLD-set. Hence,

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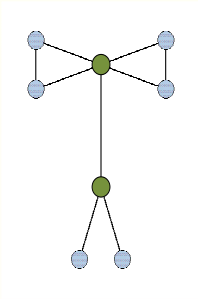
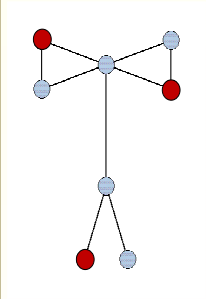
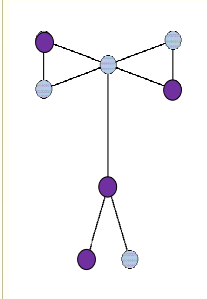
⇒ Let $S_1, S_2 \subseteq V(G)$. If S_1 is dominating and S_2 is locating, then $S_1 \cup S_2$ is an MLD-set. Hence,

$$\max\{\gamma(G), \beta(G)\} \leq \eta(G) \leq \gamma(G) + \beta(G)$$

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$\gamma(G) = 2$	$\beta(G) = 3$	$\eta(G) = 4$

$$\max\{\gamma(G), \beta(G)\} = 3 \leq \eta(G) = 4 \leq \gamma(G) + \beta(G) = 5$$

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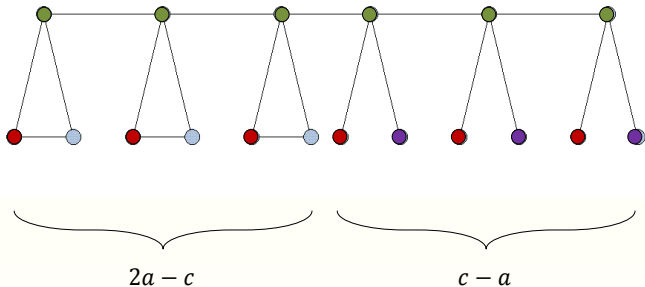
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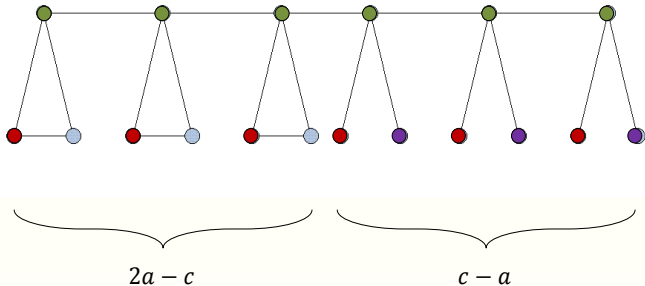
there always exists a graph G such that

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except for the case: $1 = b < a < c = a + 1$.

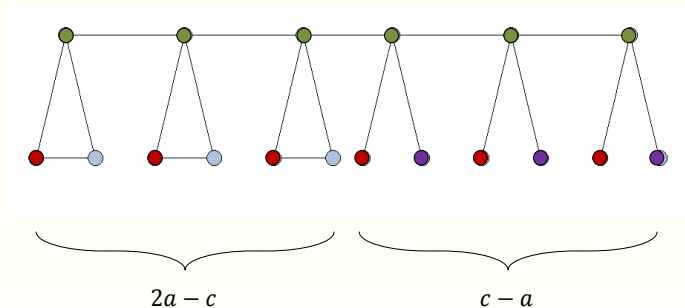


Case: $2 \leq a = b < c \leq 2a$



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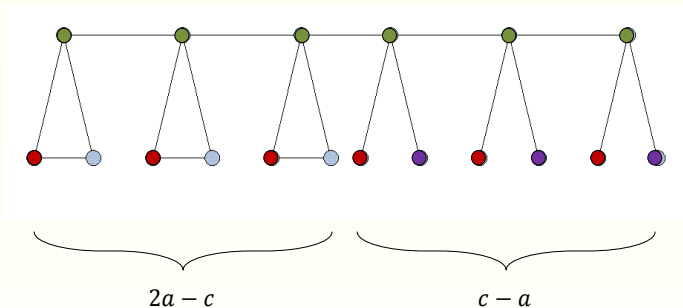
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- * Green vertices: γ -code $\rightarrow \gamma = a$
- * Red vertices: Metric basis $\rightarrow \beta = b = a$
- * Red+purple vertices: η -code $\rightarrow \eta = (2a - c) + 2(c - a) = c$

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- ▷ A set D of vertices in a graph G is is an **BLD-set**⁷ if for every two vertices $u, v \in V(G) \setminus D$,

$$\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset$$

[SI88]

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⇒ Every BLD-set is both locating and dominating. Hence,

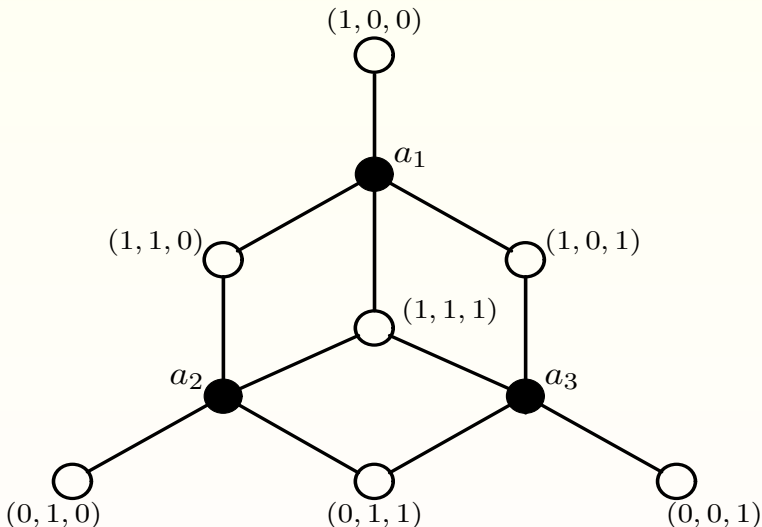
$$\max\{\gamma(G), \beta(G)\} \leq \eta(G) \leq \min\{\lambda(G), \gamma(G) + \beta(G)\}$$

and both bounds are tight.

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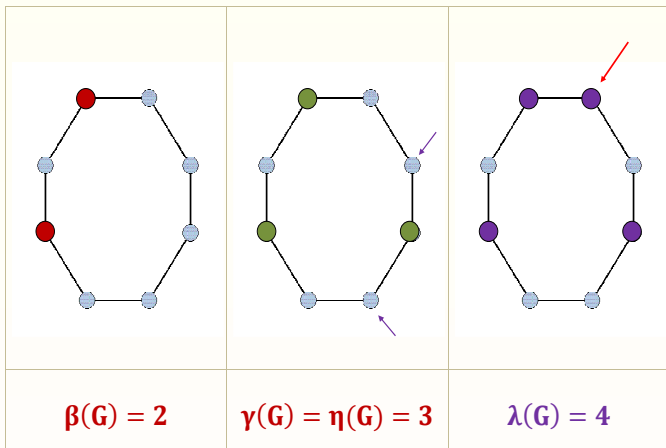
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In all cases, digit 0 means "greater than 1"

$\lambda(G) = 3$, since $\{a_1, a_2, a_3\}$ is a λ -code.



$$\max\{\gamma(G), \beta(G)\} = 3 \leq \eta(G) = 3 \leq \min\{\lambda(G), \gamma(G) + \beta(G)\} = 4$$

G	γ	β	η	λ
$P_n, n > 3$	$\lceil \frac{n}{3} \rceil$	1	$\lceil \frac{n}{3} \rceil$	$\lceil \frac{2n}{5} \rceil$
$C_n, n > 6$	$\lceil \frac{n}{3} \rceil$	2	$\lceil \frac{n}{3} \rceil$	$\lceil \frac{2n}{5} \rceil$
$K_n, n > 1$	1	$n - 1$	$n - 1$	$n - 1$
$K_{1,n-1}, n > 2$	1	$n - 2$	$n - 1$	$n - 1$
$K_{r,n-r}, n - r \geq r > 1$	2	$n - 2$	$n - 2$	$n - 2$
$W_{1,n-1}, n > 7$	1	$\lfloor \frac{2n}{5} \rfloor$	$\lceil \frac{2n-2}{5} \rceil$	$\lceil \frac{2n-2}{5} \rceil$

Domination parameters of some basic graphs

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MLD number η and BLD number λ .

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- * For every $j \neq i$, $x_j = d(x, u_j) \in \{d_{ij} - 1, d_{ij}, d_{ij} + 1\}$.

$G \neq K_{1,n-1}$ is a graph of order n , diameter $D \geq 2$ and MLD number η .

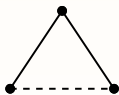
- $\eta + \lceil \frac{2D}{3} \rceil \leq n$:
- * P_{D+1} is a shortest path joining two vertices such that $d(u, v) = D$.
- * $\eta(P_{D+1}) = \lceil \frac{D+1}{3} \rceil$.
- * $\eta \leq n - (D + 1) + \lceil \frac{D+1}{3} \rceil = n - \lceil \frac{2D}{3} \rceil$.
- $n \leq \eta + \eta \cdot 3^{\eta-1}$:
- * $S = \{u_1, \dots, u_\eta\}$ is an η -set and $d(u_i, u_j) = d_{ij}$.
- * If $x = (x_1, \dots, x_\eta)$ then, for some $i \in \{1, \dots, \eta\}$, $x_i = d(x, u_i) = 1$.
- * For every $j \neq i$, $x_j = d(x, u_j) \in \{d_{ij} - 1, d_{ij}, d_{ij} + 1\}$.
- $\lambda + \lceil \frac{3D-1}{5} \rceil \leq n \leq \lambda + 2^\lambda - 1$ is similarly proved.

- $\eta(G) = 1 \Leftrightarrow \lambda(G) = 1 \Leftrightarrow G = P_2$

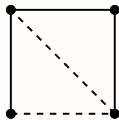
- $\lambda(G) = 2 \Rightarrow \eta(G) = 2$. [converse false]

- $\lambda = 2 \Rightarrow n \leq \lambda + 2^\lambda - 1 = 5$

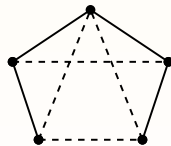
- There are 16 graphs s.t. $\lambda = 2$:



$$n = 3$$

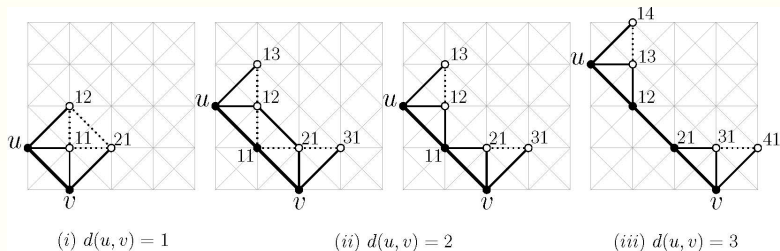


$$n = 4$$



$$n = 5$$

There are 51 graphs satisfying $\eta = 2$

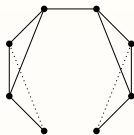
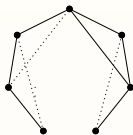
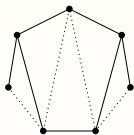
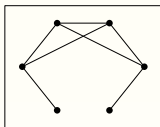
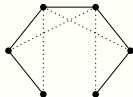
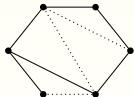
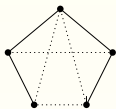


▷ $\eta = 2 \Rightarrow n \leq \eta + \eta \cdot 3^{\eta-1} = 8$

▷ Every graph verifying $\beta \leq 2$ can be embedded into the king grid.

▷ If $\{u, v\}$ is an η -set, then it is dominant, and hence $d(u, v) \leq 3$.

There are 51 graphs satisfying $\eta = 2$:



▷ For all of these graphs, $2 \leq \lambda \leq 4$.

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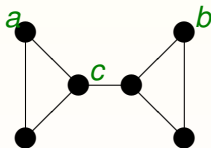
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where (see [HeOe04]) $F_1 = \{K_{r,s} : 2 \leq r \leq s\}$, etc.

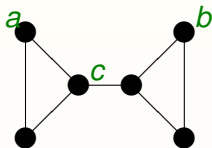
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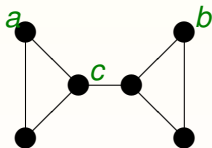


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T is a tree having $\ell(T)$ leaves, $s(T)$ support vertices, domination number $\gamma(T)$, MLD number $\eta(T)$ and BLD number $\lambda(T)$.

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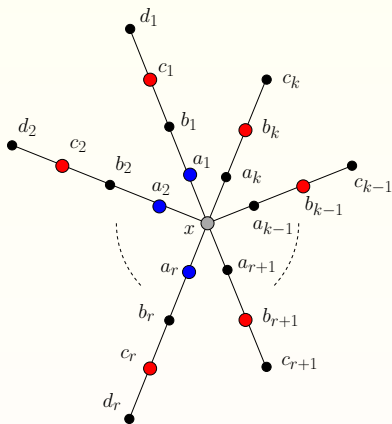
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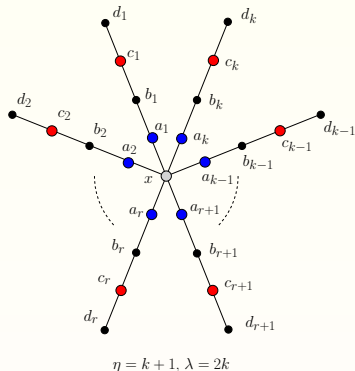
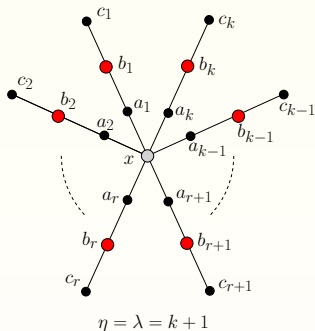
- $\eta \leq \lambda \leq 2\eta - 2$ [HeOe04]

\implies Given two integers a, b s.t. $3 \leq a \leq b \leq 2a - 2$, there always exists a tree T s.t. $\eta(T) = a$ and $\lambda(T) = b$.



$$\eta = k + 1, \lambda = k + r + 1$$

Spider with k legs, r of them having 4 edges, and the rest 3 edges.



LEFT: Spider with k legs, all of them having 3 edges.

RIGHT: Spider with k legs, all of them having 4 edges.

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- The lower bound seems not to be tight.
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\Rightarrow Given three integers a, b, c s.t. $0 < c < b < a$ and $2b - c \leq a \leq 3b - 2c - 1$, there always exists a tree T s.t. $|V(T)| = a$, $\lambda(T) = b$ and $\ell(T) - s(T) = c$.

SOME FURTHER WORK

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- The only significant result involving the Cartesian product operator is the following one:

$$\lambda(K_n \square K_m) = \begin{cases} n-1 & \text{if } 2m-1 < n \\ \lfloor \frac{2}{3}(n+m-1) \rfloor + 1 & \text{if } n \leq 2m-1, n+m = 3k+2 \\ \lfloor \frac{2}{3}(n+m-1) \rfloor & \text{in any other case} \end{cases}$$